H 1975 Q8

K.E of part $\Delta m = \frac{1}{2}\Delta m v^2$

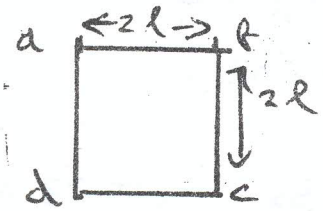
$\Rightarrow$ K.E of Body $= \sum_{\text{Body}} \left(\frac{1}{2}\Delta m v^2\right)$

$= \frac{1}{2}\sum(\Delta m v^2)$

$= \frac{1}{2}\left[\sum_{\text{Body}}(\Delta m \omega^2 x^2)\right]$ $\quad v = \omega x$

$= \frac{1}{2}\left[\sum_{\text{Body}}(\Delta m x^2)\right]\omega^2$

$= \frac{1}{2} I \omega^2$ since $I = \sum_{\text{Body}}(\Delta m x^2)$ by def^n.



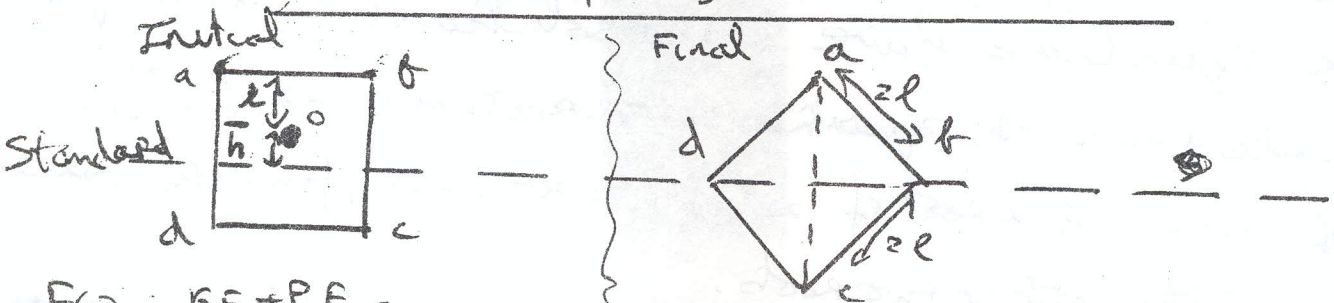
Find I_A where A is an axis through a $\perp$ to plane

$I_{ab} = \frac{4}{3}m\ell^2$] Log Book

$I_{ad} = \frac{4}{3}m\ell^2$]

$\perp$ axis theorem $\Rightarrow I_A = I_{ab} + I_{ad}$

$\Rightarrow I_A = \frac{4}{3}m\ell^2 + \frac{4}{3}m\ell^2 = \frac{8}{3}m\ell^2$ qed.



$E_{(I)}$ = K.E + P.E.

$= \frac{1}{2}I\omega^2 + mg\bar{h}_1$ [see ✱]

$= 0 + mg(\sqrt{2}\ell - \ell)$

$= mg(\sqrt{2}-1)\ell$

$E_{(II)} = \frac{1}{2}I\omega^2 + mg(0)$

$= \frac{1}{2}\frac{8}{3}m\ell^2\omega^2$

$= \frac{4}{3}m\ell^2\omega^2$.

Conservation of Energy $\Rightarrow E_{(I)} = E_{II}$

$\Rightarrow mg(\sqrt{2}-1)\ell = \frac{4}{3}m\ell^2\omega^2$

$\Rightarrow \sqrt{\frac{3g(\sqrt{2}-1)}{4\ell}} = \omega$

$\Rightarrow$ Speed of C $= \omega |AC|$

$= \sqrt{\frac{3g(\sqrt{2}-1)}{4\ell}}(2\sqrt{2}\ell)$

$= 2\sqrt{\frac{6g(\sqrt{2}-1)\ell^2}{4\ell}}$

$= \frac{2}{2}\sqrt{6g(\sqrt{2}-1)\ell}\ ms^{-1}$

Speed of C $= \sqrt{6g(\sqrt{2}-1)\ell}\ ms^{-1}$.

CALCULATIONS

✱ $\bar{h}_1 + \ell = \sqrt{2}\ell$

$\Rightarrow \bar{h}_1 = \sqrt{2}\ell - \ell$

$|AC| = \sqrt{(2\ell)^2 + (2\ell)^2}$

$= \sqrt{8\ell^2}$

$= 2\sqrt{2}\ell$.

$\Rightarrow$ a $\downarrow$ Standard position $= \sqrt{2}\ell$.